

Exercise 15

Prove the identity.

$$\sinh 2x = 2 \sinh x \cosh x$$

Solution

Use the definitions listed on page 259.

$$\begin{aligned} 2 \sinh x \cosh x &= 2 \left(\frac{e^x - e^{-x}}{2} \right) \left(\frac{e^x + e^{-x}}{2} \right) \\ &= \frac{(e^x - e^{-x})(e^x + e^{-x})}{2} \\ &= \frac{e^{2x} + 1 - 1 - e^{-2x}}{2} \\ &= \frac{e^{2x} - e^{-2x}}{2} \\ &= \sinh 2x \end{aligned}$$